

EUF - 2013 Zsem

12/03/2016

Q1. a) Poter in fio:

$$E = \frac{\lambda \cdot k}{2\pi r \cdot X} = \frac{\lambda}{2\pi r \epsilon_0}$$



$$r = \sqrt{(x-a)^2 + y^2 + z^2}$$

$$V = \int_a^r \frac{\lambda}{2\pi r \epsilon_0} = -\frac{\lambda}{2\pi \epsilon_0} \ln\left(\frac{r}{a}\right) = \frac{\lambda}{2\pi \epsilon_0} \ln\left(\frac{\sqrt{(x-a)^2 + y^2 + z^2}}{a}\right)$$

b) Consider also fio in $x = -a$ with $-\lambda$.

$$V_-(x, y, z) = \frac{(-\lambda)}{2\pi \epsilon_0} \ln\left(\frac{r_-}{a}\right)$$

$$V_T = V_+ + V_- = \frac{\lambda}{2\pi \epsilon_0} \left[\ln\left(\frac{r_+}{a}\right) - \ln\left(\frac{r_-}{a}\right) \right] = \frac{\lambda}{2\pi \epsilon_0} \ln\left(\frac{r_+}{r_-}\right)$$

$$r_- = \sqrt{(x+a)^2 + y^2 + z^2}$$

$$V(x, y, z) = \frac{\lambda}{2\pi \epsilon_0} \ln\left(\frac{\sqrt{(x+a)^2 + y^2 + z^2}}{\sqrt{(x-a)^2 + y^2 + z^2}}\right)$$

c) surface charge σ inside:

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial n} \Rightarrow \sigma = -\epsilon_0 \frac{\partial V}{\partial x} \Big|_{x=0} = -\frac{\lambda}{4\pi} \left[\frac{2(x+a)}{(x+a)^2 + y^2 + z^2} - \frac{2(x-a)}{(x-a)^2 + y^2 + z^2} \right] \Big|_{x=0}$$

$$\sigma = -\frac{\lambda}{4\pi} \left[\frac{4a}{a^2 + y^2 + z^2} \right] = -\frac{\lambda a}{\pi(a^2 + y^2 + z^2)}$$

$$d) \int_{-\infty}^{\infty} \frac{\lambda a}{\pi(a^2+y^2+z^2)} dy = a_1 = a^2+z^2 = \frac{\lambda a}{\pi} \int_{-\infty}^{\infty} \frac{1}{a^2+y^2} dy = \frac{\lambda a}{\pi(a^2+z^2)} \cdot \arctan\left(\frac{y}{a^2+z^2}\right)$$

$$= \frac{\lambda a}{\pi(a^2+z^2)} \cdot \left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) \right] = \frac{\lambda a}{a^2+z^2} \quad ?$$

Q2. a) $\vec{L} = \vec{r} \times \vec{p} \Rightarrow \tau = \frac{dL}{dt} = \vec{r} \times \vec{F} =$

$$\vec{\tau} = \int \vec{r} \times d\vec{p} = - \frac{d\vec{p}}{dt} = - \frac{d}{dt} \int \vec{p} da = -\pi b^2 \frac{d\vec{p}}{dt}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = a \lambda \vec{e}_\perp dl \Rightarrow \tau = a \lambda \int \vec{e}_\perp dl = -\pi b^2 a \lambda \frac{d\vec{p}}{dt}$$

$$L = \int_{B_0}^0 \tau dt = \boxed{+\pi b^2 a \lambda B_0}$$

b) if $\omega = \frac{v}{\lambda} \Rightarrow \lambda = \frac{2\pi a}{\omega} \cdot \text{rev} = \frac{2\pi a}{\omega} \cdot \left(\frac{1}{T}\right)$



$$B = \frac{\mu_0}{4\pi} \int \frac{d\vec{l} \times \vec{r}}{r^2} = \frac{\mu_0 I}{4\pi a^2} \int \frac{d\vec{l}}{a^2} = \frac{\mu_0 I}{4\pi a^2} \cdot 2\pi a = \frac{\mu_0 I}{2a}$$

$$d\vec{l} \times \vec{r} = dl \cdot \sin 90^\circ$$

$$L = I \cdot \omega$$

$$\omega = \frac{L}{I}$$

$$\lambda = \frac{2\pi a^2 b^2}{I} \lambda B_0 \Rightarrow \boxed{B = \frac{\mu_0 a \pi b^2 \lambda B_0}{I}}$$

EVF 2013, 25m pt 2

Q3.

$$a) E = h\nu \Rightarrow \nu = \frac{c}{\lambda} = \frac{3 \cdot 10^8}{480 \cdot 10^{-9}} = 0,6 \cdot 10^{15}$$

$$I = 10 \frac{W}{m^2} \Rightarrow P = I \cdot A = 10 \frac{W}{m^2} \cdot 10^{-4} = 10^{-3} W \left(\frac{J}{s} \right)$$

$$E = h\nu = 6,63 \cdot 10^{-34} \cdot 0,6 \cdot 10^{15} J = 3,97 \cdot 10^{-19} J$$

$$n = 4,14 \cdot 10^{-19} \cdot 0,6 \cdot 10^{15} eV = \boxed{2,48 eV}$$

$$b) \frac{P}{E} = n = \frac{10^{-3} \left[\frac{J}{s} \right]}{3,97 \cdot 10^{-19} \left[\frac{J}{photon} \right]} = 1,22 \cdot 10^{16} \frac{photons}{s}$$

$$c) 20\% \Rightarrow 2,24 \cdot 10^{16} \frac{electrons}{s} = I$$

$$I = 0,14 \cdot 10^{16} \cdot e$$

$$K = h\nu - W$$

$$K_{max} = h\nu - W_0 \rightarrow \text{work function}$$

$$d) 2,2 = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{4,14 \cdot 10^{-15} \cdot 3 \cdot 10^8}{2,2} = 6 \cdot 10^{-7} = 600 nm$$

$$94. U = \frac{m\omega^2 x^2}{2}, \psi(x) = A e^{-bx^2}$$

↳ ground state

$$a) \quad -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi + \frac{m\omega^2}{2} x^2 \psi = E A e^{-bx^2}$$

$$x = \frac{m\omega}{\hbar}$$

$$y = \sqrt{x} = \frac{m\omega}{\hbar} x^2$$

$$b) \quad -\frac{\hbar^2}{2m} \cdot A (4b^2 x^2 e^{-bx^2}) + \frac{m\omega^2}{2} x^2 A e^{-bx^2} = E A e^{-bx^2}$$

$$-\frac{\hbar^2}{2m} 4b^2 x^2 + \frac{m\omega^2}{2} x^2 = E$$

$$b = \frac{m\omega}{4\hbar^2} = \frac{m\omega}{2\hbar}$$

$$\frac{\hbar^2}{m} 2b^2 x^2 = \frac{m\omega^2}{2} x^2 - 2E$$

$$b = \sqrt{\frac{m\omega^2 x^2 - 2E}{4\hbar^2 x^2}} = \frac{m\omega}{2\hbar} - \frac{\sqrt{2E}}{2\hbar x}$$

$$\sqrt{\frac{m\omega}{4\hbar x^2}}$$

why $E=0$?

$$\hat{H}\psi = E\psi$$

$$E = \left(\frac{1}{2} + n\right)\hbar\omega = \frac{\hbar\omega}{2}$$

$$c) \int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1 \Rightarrow A^2 \int_{-\infty}^{\infty} e^{-2bx^2} dx = 1 \Rightarrow A^2 \left(\frac{\pi}{2b}\right)^{\frac{1}{2}} = 1$$

$$d) \int_{-x_{max}}^{x_{max}} |\psi(x)|^2 dx = \int_{-\left(\frac{\hbar}{m\omega}\right)^{\frac{1}{2}}}^{\left(\frac{\hbar}{m\omega}\right)^{\frac{1}{2}}} A^2 e^{-2bx^2} dx$$

$$A = \frac{1}{\left(\frac{\hbar}{m\omega}\right)^{\frac{1}{4}}}$$

$$A = \left(\frac{m\omega}{\hbar}\right)^{\frac{1}{4}}$$

other way to do it!

EVF 2013 - 28m

13/03/2016

Q4.

$$a) \frac{-\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + \frac{1}{2} \mu x^2 \psi(x) = E \psi(x)$$

$$\psi'' + \left(\frac{\mu x^2}{\hbar^2} + \frac{2mE}{\hbar^2} \right) \psi = 0 \quad \alpha = \frac{\mu x}{\hbar^2}, \quad \lambda = \frac{2mE}{\hbar^2}$$

$$\xi = \alpha^{\frac{1}{2}} x \quad d\xi = \alpha^{\frac{1}{2}} dx \Rightarrow \psi''(\xi) + \left(\frac{\lambda}{\alpha} - \frac{\alpha^2 \xi^2}{\alpha} \right) \psi = 0$$

$$\psi''(\xi) - \xi^2 \psi(\xi) = 0 \quad \text{for } \xi \rightarrow \pm \infty$$

$$\left(\frac{d^2}{d\xi^2} - \xi^2 \right) \psi = \left(\frac{d}{d\xi} - \xi \right) \left(\frac{d}{d\xi} + \xi \right) \psi(\xi) = 0$$

$$\psi'(\xi) + \xi \psi(\xi) = 0$$

$$\psi(\xi) = A e^{-\frac{\xi^2}{2}}$$

$$\Rightarrow E = \frac{\lambda \hbar^2 \alpha}{2m} = \frac{1}{2} \hbar \sqrt{\frac{\mu}{m}} = \frac{1}{2} \hbar \omega$$

QS. $P_0 = 3P_0$

$$a) U = \frac{3}{2} NRT + \frac{3}{2} PV, \quad S = \frac{3}{2} N \ln T + P_0 V + c, \quad \text{with } dS = 0$$

$$\Delta S = R \ln \Delta T + R \ln \Delta V + c \quad \Delta T = \Delta V \Rightarrow NR = \Delta P$$

$$NR = 3P_0 X$$

$$P = \frac{NR}{21}$$

$$NR = PV \Rightarrow NR \Delta T = \Delta P \Delta V$$

(5)

$$\Delta S = 0$$

$$\Delta S = 0$$

$$\frac{3}{2} R \ln T + R \ln V = C$$

$$\ln T^{\frac{3}{2}} + \ln V = C$$

$$\ln(T^{\frac{3}{2}} \cdot V) = C$$

$$\ln\left(\frac{\Delta P^{\frac{3}{2}} \cdot \Delta V^{\frac{5}{2}}}{(NR)^{\frac{3}{2}}}\right) = C$$

$$\Delta T = \frac{\Delta P \Delta V}{NR}$$

$$(\ln(P^{\frac{3}{2}} V^{\frac{5}{2}}) - 3 \ln(NR)) = C$$

$$P^{\frac{3}{2}} V^{\frac{5}{2}} = \text{const}$$

$$b) \quad P_i^{\frac{3}{2}} V_i^{\frac{5}{2}} = P_f^{\frac{3}{2}} V_f^{\frac{5}{2}} \quad V_i = V_0$$

$$V_f^{\frac{5}{2}} \cdot (32 P_0)^{\frac{3}{2}} = P_0^{\frac{3}{2}} V_0^{\frac{5}{2}} \quad P_f = 32 P_0$$

$$V_f = \frac{V_0}{32^{\frac{2}{5}}}$$

$$\Delta V_f = \frac{V_0}{32^{\frac{2}{5}}} + V_0$$

$$B$$

$$V_f = V_0 + \Delta V_f^A$$

$$V_f = \frac{V_0}{32^{\frac{2}{5}}} + 2V_0$$

$$c) \quad A$$

$$NRT_0 = P_0 V_0$$

$$NRT_f = 32 P_0 \cdot \frac{V_0}{32^{\frac{2}{5}}}$$

$$T_f = T_0 \cdot 32^{\frac{2}{5}}$$

$$B$$

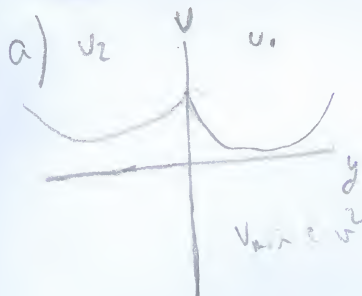
$$NRT_f = P_f V_0 \left(2 - \frac{1}{32^{\frac{2}{5}}}\right)$$

$$d) \quad dV = dQ - dW$$

EVF 2013, 25%

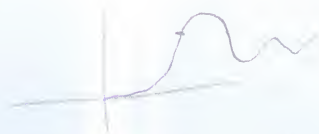
$r_{min} \rightarrow E$

Q6.



$$E_1 = \frac{mv_1^2}{2} + U_1 \Rightarrow v_1 = \sqrt{E - U_1}$$

$$E_2 = \frac{mv_2^2}{2} + U_2$$



b) $E_2 = E_1$

$$v_2 = \sqrt{\left(\frac{mv_1^2}{2} + U_1 - U_2\right) \cdot \frac{2}{m}}$$

$v_{min} \Rightarrow v_2 = 0$

$$\frac{mv_1^2}{2} + U_1 - U_2 = 0 \Rightarrow v_{min} = \sqrt{(U_2 - U_1) \cdot \frac{2}{m}}$$

c) $\sin \theta_1 = \frac{x_1}{v_1}$

$$\dot{x} = \sqrt{\left(\frac{1}{2} \dot{x}^4 - c x^2 - E\right) \cdot \frac{2}{m}}$$

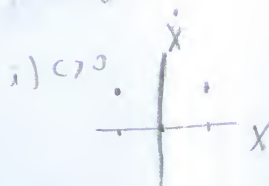
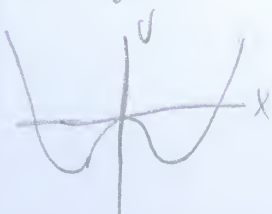
Q7. $V(x) = \frac{1}{2} x^4 - c x^2$

$$E = \frac{m \dot{x}^2}{2} + \frac{1}{2} x^4 - c x^2 \Rightarrow \dot{x} = \sqrt{\frac{2}{m} \left(\frac{1}{2} x^4 - c x^2 - E \right)}$$

$$\dot{x} = \sqrt{\frac{1}{m} \left(x^4 - 2c x^2 - 2E \right)}$$

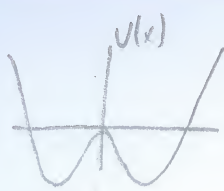
$$0 = \frac{1}{2} \left(\frac{x^4}{m} - \frac{2c x^2}{m} - \frac{2E}{m} \right)^{-\frac{1}{2}}$$

c)

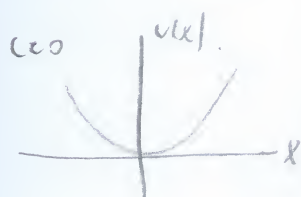
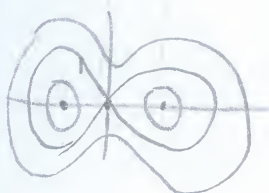


(7)

c7)



$C=0$



$$b) E = \frac{mv^2}{2} + \frac{1}{2}x^4 - cx^2$$

soluções periódicas que circundam o ponto fixo 0

$$c) E = \frac{mv^2}{2} + \frac{1}{2}x^4 \Rightarrow v^2 = \frac{2E}{m} - \frac{1}{2m}x^4$$

Q8 $\left(\frac{\vec{p}^2}{2m} - c v_p^2 \right) \psi(p, t) = i \frac{\partial}{\partial t} \psi(p, t)$

$$\psi(p, t) = \frac{1}{(2\pi\hbar)^{3/2}} \int d^3r e^{-i\vec{p}\vec{r}/\hbar} \psi(r)$$

$$\psi(r) = \frac{1}{(2\pi\hbar)^{3/2}} \int d^3p e^{i\vec{p}\vec{r}/\hbar} \psi(p)$$

$$i \frac{d}{dt} \int \frac{1}{C} d^3p e^{i\vec{p}\vec{r}/\hbar} \psi(p) = \frac{1}{C} \int d^3p e^{i\vec{p}\vec{r}/\hbar} \left(\frac{\partial \psi(p)}{\partial t} \right)$$

$$= \frac{1}{C} \int d^3p \exp\left(i\frac{\vec{p}\vec{r}}{\hbar}\right) \cdot \left[\frac{\vec{p}^2}{2m} - c v_p^2 \right] \psi(p)$$

$$U = \frac{p^2}{2m} \Rightarrow dU = \frac{p}{m} dp \Rightarrow \frac{p^2}{2m} \cdot \frac{1}{r} \exp\left(i\frac{\vec{p}\vec{r}}{\hbar}\right) = \int \frac{p}{m} \frac{1}{r} \exp\left(i\frac{\vec{p}\vec{r}}{\hbar}\right) dp$$

$$dU = \exp\left(i\frac{\vec{p}\vec{r}}{\hbar}\right)$$

$$U = \exp\left(i\frac{\vec{p}\vec{r}}{\hbar}\right) \cdot \frac{1}{r}$$

$$U = \frac{p}{m} \quad dU = \frac{1}{m}$$

$$dU = \frac{1}{m} \exp\left(i\frac{\vec{p}\vec{r}}{\hbar}\right) \Rightarrow U = \left(\frac{1}{m}\right) \exp\left(i\frac{\vec{p}\vec{r}}{\hbar}\right)$$

$$\frac{p}{m} \frac{1}{r} \exp\left(i\frac{\vec{p}\vec{r}}{\hbar}\right) + \frac{p}{m} \cdot \left(\frac{1}{r}\right)' \exp\left(i\frac{\vec{p}\vec{r}}{\hbar}\right)$$

Q5

$$a) [\hat{S}_x, \hat{S}_y] = \hat{S}_x \hat{S}_y - \hat{S}_y \hat{S}_x = \frac{\hbar^2}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \frac{\hbar^2}{2} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \frac{\hbar^2}{2} \begin{pmatrix} i & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & -i \end{pmatrix} - \frac{\hbar^2}{2} \begin{pmatrix} -i & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & i \end{pmatrix} = \frac{\hbar^2}{2} \begin{pmatrix} 2i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2i \end{pmatrix} = i\hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$[\hat{S}_y, \hat{S}_z] = 2i\hbar$ in permutation cycle

$= i\hbar \hat{S}_x$

b) $\hat{S}_z |\psi\rangle = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \hbar \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

eigenvalues $\rightarrow \hbar, -\hbar$ (each twice)

$\hbar, -\hbar$

$\begin{bmatrix} \hbar & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \hbar \begin{bmatrix} a \\ b \\ c \end{bmatrix} \Rightarrow \begin{matrix} \hbar a = \hbar a \\ 0 = \hbar b \\ -\hbar c = \hbar c \end{matrix} \Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\hbar, -\hbar$

$\hbar a = -\hbar a \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad \hbar a = 0 \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$0 = -\hbar b$

$-\hbar c = \hbar c$

$P\left(\begin{smallmatrix} 1 \\ 0 \\ 0 \end{smallmatrix}\right) = \frac{1}{6}; \quad P\left(\begin{smallmatrix} 0 \\ 1 \\ 0 \end{smallmatrix}\right) = \frac{4}{6}, \frac{2}{3}$

c) $|\psi\rangle = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = 1|a\rangle + 1|b\rangle + 2|c\rangle$

$A(1+1+4) = 1 \Rightarrow \frac{1}{\sqrt{6}} (1|a\rangle + 1|b\rangle + 2|c\rangle) \quad \sqrt{d} \quad ?$

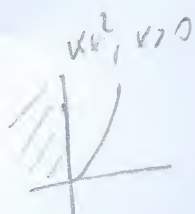
$A = \sqrt{\frac{1}{6}}$

ЕУР 2013, 25с

17/3/2014

Q10.

a)



$$\epsilon_n = (n + \frac{1}{2}) \hbar \omega \quad n = 1, 3, 5, \dots$$

ψ_n is parabolic

ψ_n is parabolic

$$\hat{p} \psi(x) = \psi(-x) = 0$$

$$b) z = \sum e^{-p \epsilon_n} = e^{-\frac{p \hbar \omega}{2}} \cdot \sum e^{-p \hbar \omega n} = \sum (e^{-p \hbar \omega})^n = e$$